The Menger Sponge

As mentioned earlier fractals are formed by fitting together geometric patterns that repeat but assemble based on specific methods. Such regular, repeatable methods are known as Algorithms. Formulas in general can also be seen as algorithms.

The following algorithm builds a **Menger Sponge (or Cube)**, a self-similar 3D fractal cube.

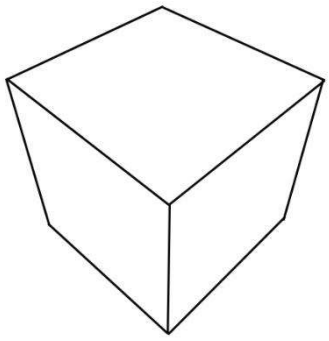
A few unusual and unique properties emerge from the construction of the Menger Sponge:

(i) the resulting cube has a near zero volume, and
(ii) the resulting cube has infinite surface.

The process and results are explained as follows.

Step 1:

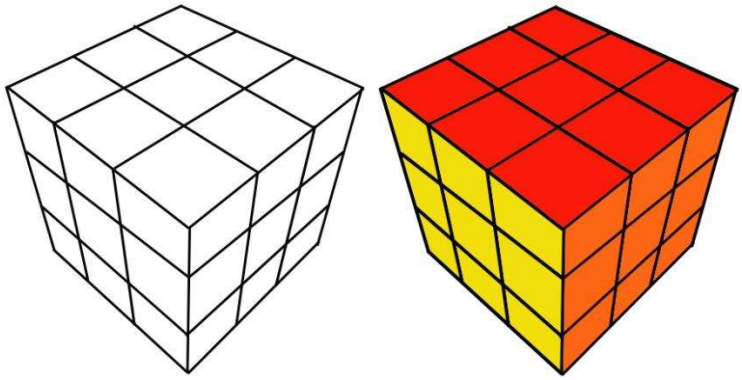
Start with an empty cube.



Step 2:

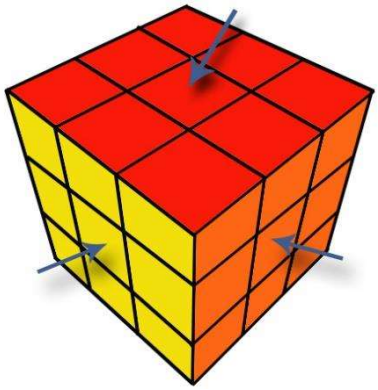
Divide the cube into 27 smaller cubes, by dividing each face of the cube into 9 smaller squares, as shown below.

If you were to add different color to each face, it would look a bit like a Rubik cube (below, right). Let's keep the color to help us track the changes.



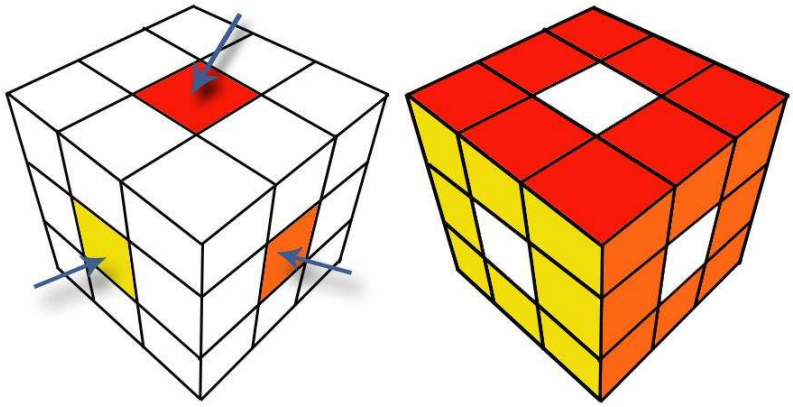
Step 3:

Focus on the cells at the center of each face, as pointed by the blue arrows. Each has a smaller cube residing in that position.



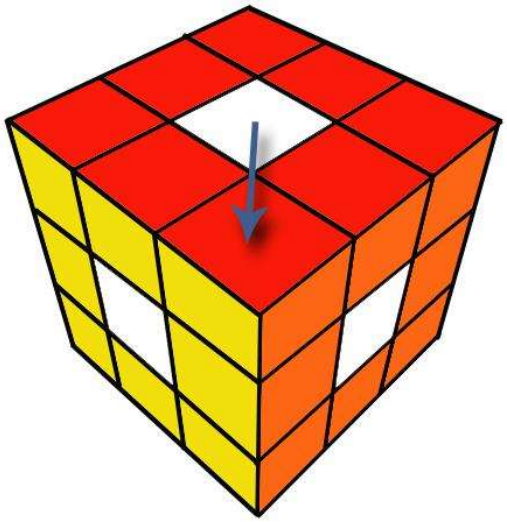
Step 4:

Cut out, or perforate, the center cubes pointed by the arrows in the middle. The cube on the right shows the missing parts in white.



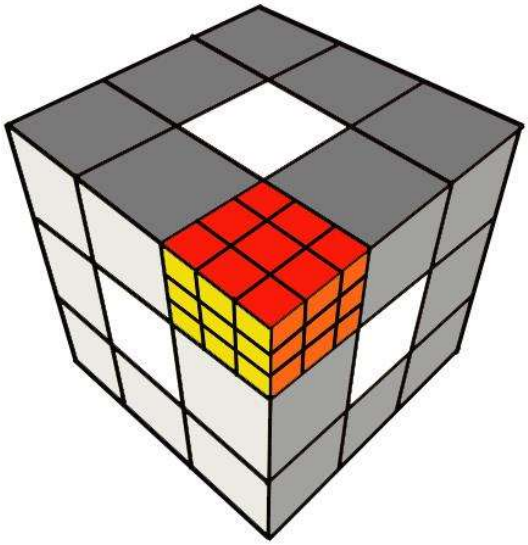
Step 5:

Pick one of the small colored cubes, for example the one pointed by arrow in the front top corner.

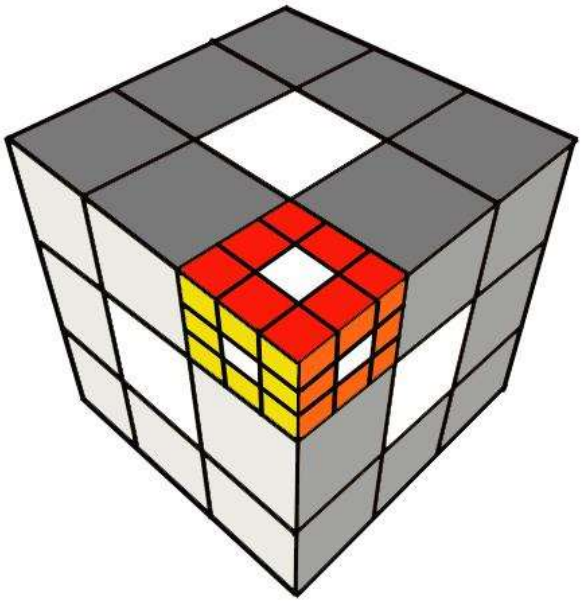


Step 6:

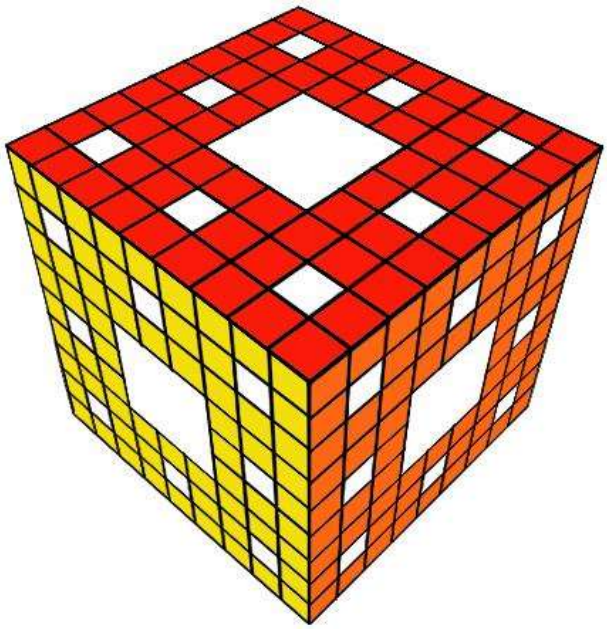
Repeat Step 2 on the selected cube.



Then follow the methods in steps 3 and 4 as well. A series of new perforations are made as shown below in white.

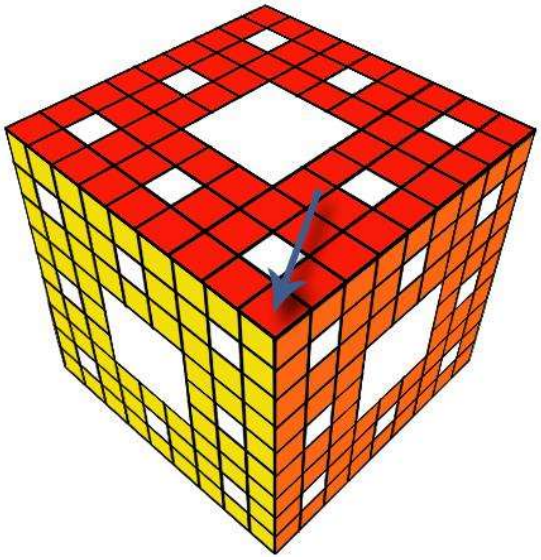


Repeat the process on every one of the remaining grey cubes. Do this on every side. The results are should look like this:



Step 7:

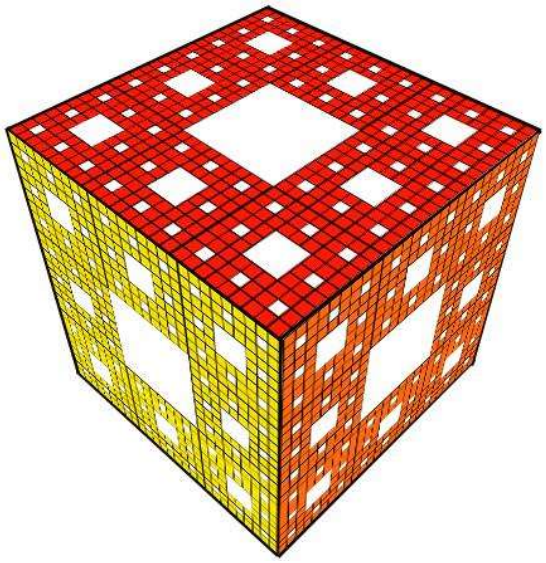
Using the current cube, return to Step 5 and repeat from there on an even smaller cube all the way to Step 6. Just like the previous time, pick the top front corner cube, where the arrow points below.



Repeat Step 7 as many times as you can, then move on to Step 8.

Step 8:

Decide when the cubes have become too small to continue the process, then stop.



Conclusions

The following figure shows that many cubes of different sizes were removed throughout this algorithm, initially larger, then smaller, and we stopped when they were too small.